Comparison of single and multiple flow direction algorithms for computing topographic parameters in TOPMODEL

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Abstract. Single flow direction (sfd) and multiple flow direction (mfd) algorithms were used to compute the spatial and statistical distributions of the topographic index used in the watershed model TOPMODEL. An sfd algorithm assumes that subsurface flow occurs only in the steepest downslope direction from any given point; an mfd algorithm assumes that subsurface flow occurs in all downslope directions from any given point. The topographic index in TOPMODEL is \( \ln (a/\tan \beta) \), where \( \ln \) is the Napierian logarithm, \( a \) is the upslope area per unit contour length, and \( \tan \beta \) is the slope gradient. The \( \ln (a/\tan \beta) \) distributions were computed from digital elevation model (DEM) data for locations with diverse topography in Arizona, Colorado, Louisiana, Nebraska, North Carolina, Oregon, Pennsylvania, Tennessee, Vermont, and Virginia. The means of the \( \ln (a/\tan \beta) \) distributions were higher when the mfd algorithm was used for computation compared to when the sfd algorithm was used. The variances and skews of the distributions were lower for the mfd algorithm compared to the sfd algorithm. The differences between the mfd and sfd algorithms in the mean, variance, and skew of the \( \ln (a/\tan \beta) \) distribution were almost identical for the various DE\( M \)s and were not affected by DEM resolution or watershed size. TOPMODEL model efficiency and simulated flow paths were affected only slightly when the \( \ln (a/\tan \beta) \) distribution was computed with the sfd algorithm instead of the mfd algorithm. Any difference in the model efficiency and simulated flow paths between the sfd and mfd algorithms essentially disappeared when the model was calibrated by adjusting subsurface hydraulic parameters.

Introduction

Topography is recognized as an important factor in determining the streamflow response of watersheds to precipitation [Kirkby and Chorley, 1967; Dunne et al., 1975; O'Loughlin, 1981; Beven and Wood, 1983]. Topography defines the effects of gravity on the movement of water in a watershed and therefore influences many aspects of the hydrologic system. For example, topography has been shown to affect the flow path that precipitation follows before it becomes streamflow [Wolock et al., 1990], the spatial distribution of soil moisture within a watershed [Burt and Butcher, 1985], and the chemical characteristics of streamflow [Wolock et al., 1989, 1990].

TOPMODEL [Beven and Kirkby, 1979] is a topography-based watershed hydrology model that has been used to study a range of topics, including flood frequency derivation [Beven, 1986a, b, 1987], model parameter calibration [Hornberger et al., 1985], carbon budget simulation [Band et al., 1991; Hornberger et al., 1994], spatial scale effects on hydrologic processes [Beven et al., 1988; Famiglietti and Wood, 1991; Sivapalan et al., 1987, 1990; Wood et al., 1988, 1990], topographic effects on water quality [Wolock et al., 1989, 1990], topographic effects on streamflow [Beven and Wood, 1983; Beven et al., 1984; Kirkby, 1986; Quinn et al., 1991], climate change effects on hydrologic processes [Wolock and Hornberger, 1991], the geomorphic evolution of basins [Ijiasz-Vasquez et al., 1992], the identification of hydrologic flow paths [Robson et al., 1992], and the effects of digital elevation data resolution on model predictions [Quinn et al., 1995; Wolock and Price, 1994; Zhang and Montgomery, 1994].

Application of TOPMODEL requires computation of the frequency or spatial distribution of a topographic index from topographic data, such as a contour map or a digital elevation model (DEM). This topographic index, which can be computed at any location \( x \) in the watershed, is \( \ln (a/\tan \beta) \), where \( \ln \) is the Napierian logarithm, \( a \) is the upslope area per unit contour length (sometimes called the specific catchment area) from location \( x \), and \( \tan \beta \) is the slope gradient at \( x \). It is assumed in TOPMODEL that the spatial distribution of \( \ln (a/\tan \beta) \approx \) \( \beta \) approximates the spatial distribution of the depth to the water table in a watershed. Tests of this assumption have produced mixed results [e.g., Barling et al., 1994; Troch et al., 1993], but TOPMODEL continues to be used widely. (See Beven et al. [1994] for a comprehensive discussion of TOPMODEL limitations and applications.)

In the earlier applications of TOPMODEL [e.g., Beven and Kirkby, 1979; Beven and Wood, 1983] the frequency distribution of \( \ln (a/\tan \beta) \) was derived manually from contour maps. Later applications of the model used computer software and DE\( M \)s to derive the spatial and statistical distributions of \( \ln (a/\tan \beta) \). Two types of software algorithms have been used to compute the \( \ln (a/\tan \beta) \) distribution from DE\( M \)s: a single flow direction (sfd) algorithm and a multiple flow direction (mfd) algo-
rithm. In the sfd algorithm it is assumed that subsurface flow at every point occurs only in the steepest downslope direction from any given point. In the mfd algorithm it is assumed that subsurface flow at every point occurs in all downslope directions from any given point. Quinn et al. [1991] showed that the spatial patterns and frequency distributions of \( \ln(\alpha/\tan \beta) \) computed using an sfd algorithm were different from those obtained when an mfd algorithm was used for the Borotou watershed in the Ivory Coast. The authors suggested that the mfd algorithm produced a more realistic looking spatial pattern of \( \alpha \) on the hillslopes of the watershed. They also speculated that the two algorithms would result in different TOPMODEL predictions. Similar findings were reported by Moore [1995] for the Coweeta watershed in North Carolina. In addition, Moore found that compared to the sfd algorithm, the mfd algorithm resulted in a higher mean \( \ln(\alpha/\tan \beta) \) distribution.

In this paper the effects of sfd and mfd algorithms on \( \ln(\alpha/\tan \beta) \) distributions and TOPMODEL are examined in detail. The specific objectives are (1) to compare the \( \ln(\alpha/\tan \beta) \) distributions computed using sfd and mfd algorithms for several diverse topographic settings and (2) to determine how differences in the \( \ln(\alpha/\tan \beta) \) distributions affect hydrologic characteristics simulated by TOPMODEL. The first objective is accomplished by comparing \( \ln(\alpha/\tan \beta) \) distributions computed from DEMs in Arizona, Colorado, Louisiana, Nebraska, North Carolina, Oregon, Pennsylvania, Tennessee, Vermont, and Virginia. The second objective is accomplished by applying TOPMODEL to watersheds in Vermont and Virginia.

**Description of TOPMODEL**

TOPMODEL simulates the variable source area concept of watershed hydrology [Beven and Kirkby, 1979]. (See Beven et al. [1994] or Wolock [1993] for a complete description of TOPMODEL.) The variable source area concept states that overland flow is produced only over a small fraction of the total watershed area. The land surface areas that produce overland flow are those that become saturated during precipitation events; they occur where the water table rises to the land surface. Overland flow is produced when precipitation falls on a saturated land surface area or when subsurface flow returns to the land surface and flows overland [Dunne et al., 1975]. The saturated land surface areas (called source or contributing areas) are variable in that they contract and expand over parts of the watershed. The dynamics of the saturated land surface areas are controlled by watershed topographic and subsurface hydraulic characteristics and the state of wetness of the watershed. The state of wetness of the watershed changes over time as a function of the relative balance between input (precipitation and output (evapotranspiration, overland flow, and subsurface flow).

The mathematical starting points used to derive the fundamental TOPMODEL equations are (1) the continuity equation, (2) Darcy's law, and (3) the assumption that saturated hydraulic conductivity decreases exponentially with soil depth. (The validity of the third starting point is supported by data of Beven [1984] and Elsenbeer et al. [1992].) One fundamental equation in TOPMODEL gives the relation of the depth to the water table at any location \( x \) (\( z_{wt}(x) \)) to the watershed average value of the depth to the water table (\( \bar{z}_{wt} \)), a parameter related to the rate of change of saturated hydraulic conductivity with depth (\( f \)), the watershed area upslope from location \( x \) per unit contour length at \( x \) (\( \alpha \)), the gravitational gradient at location \( x \) (\( \tan \beta \), where \( \beta \) is the land surface slope), and the watershed mean value of \( \ln(\alpha/\tan \beta) \) (\( \lambda \)), where \( \ln \) is the Napierian logarithm:

\[
z_{wt}(x) = \bar{z}_{wt} + \left(1/f\right) \ln \left(\alpha/\tan \beta\right).
\]

Equation (1) is applied to every location \( x \) in the watershed at every time step to determine the location and extent of saturated land surface areas, indicated by locations where \( z_{wt}(x) \leq 0 \). The fraction of the land surface area that is saturated is multiplied by the precipitation intensity (an observed input) to compute the amount of overland flow. Equation (1) shows that values of \( \ln(\alpha/\tan \beta) \) relate directly to the topographic likelihood of development of saturated, overland flow-producing source areas; higher values of \( \ln(\alpha/\tan \beta) \) indicate greater potential for development of saturation. High values of \( \ln(\alpha/\tan \beta) \) occur at locations where large upslope areas are drained (high value of \( \alpha \)) and where the local gravitational gradient is low (low value of \( \tan \beta \)).

Another important equation shows the relation of total watershed subsurface flow \( q_{subsurface} \) to the watershed average value of the maximum saturated hydraulic conductivity \( (K_0) \), the parameter related to the rate of change of saturated hydraulic conductivity with depth (\( f \)), the watershed average value of the depth to the water table (\( \bar{z}_{wt} \)), and the watershed mean value of \( \ln(\alpha/\tan \beta) \) (\( \lambda \)):

\[
q_{subsurface} = (K_0 f) e^{-\lambda} e^{-f \bar{z}_{wt}}.
\]

Equation (2) shows that \( \lambda \), the mean of the \( \ln(\alpha/\tan \beta) \) distribution, is inversely related to the potential rate of subsurface flow; watersheds with high \( \lambda \) values have low potential subsurface flow rates.

It was assumed in this study that all locations in a watershed with the same \( \ln(\alpha/\tan \beta) \) value were hydrologically similar. This assumption usually is made when using TOPMODEL [e.g., Beven and Kirkby, 1979; Hornberger et al., 1985; Wolock et al., 1989, 1990], and it allows the aggregation of the \( \ln(\alpha/\tan \beta) \) distribution from a spatially explicit description of the watershed into one composed of intervals of \( \ln(\alpha/\tan \beta) \). The model equations that relate hydrologic characteristics to topography do not change, but the calculations are performed using the \( \ln(\alpha/\tan \beta) \) values of frequency distribution interval midpoints instead of the individual spatially distributed values. By knowing the relative frequency (i.e., the proportion of watershed area) corresponding to each interval midpoint, total watershed values for the modeled hydrologic states can be calculated. The frequency distribution can be computed from the mean, variance, and skew values of the \( \ln(\alpha/\tan \beta) \) spatial distribution.

**Digital Elevation Model Data**

This study used DEMs from Arizona, Colorado, Louisiana, Nebraska, North Carolina, Oregon, Pennsylvania, Tennessee, Vermont, and Virginia (Table 1). The DEMs were derived from U.S. Geological Survey (USGS) 1:250,000-scale topographic maps, USGS 1:24,000-scale maps, or independent topographic surveys. The 1:250,000-scale DEMs are on a 3-arc sec (about 90-m) grid [U.S. Geological Survey, 1987] and were projected into universal transverse Mercator metric units using the geographic information system package ARC/INFO GRID, version 6.1.1 (Environmental Systems Research Insti-
Table 1. Digital Elevation Models (DEMs) Used to Compute In (a/tan β) Distributions

<table>
<thead>
<tr>
<th>Location</th>
<th>Source</th>
<th>Grid Resolution, m</th>
<th>Mean Elevation, m above sea level</th>
<th>Range in Elevation, m</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phoenix, Arizona</td>
<td>1:250,000</td>
<td>94.2</td>
<td>477</td>
<td>602</td>
<td>USGSa</td>
</tr>
<tr>
<td>Denver, Colorado</td>
<td>1:250,000</td>
<td>92.8</td>
<td>2,346</td>
<td>968</td>
<td>USGSa</td>
</tr>
<tr>
<td>Baton Rouge, Louisiana</td>
<td>1:250,000</td>
<td>93.4</td>
<td>34</td>
<td>100</td>
<td>USGSa</td>
</tr>
<tr>
<td>Broken Bow, Nebraska</td>
<td>1:250,000</td>
<td>92.9</td>
<td>611</td>
<td>238</td>
<td>USGSa</td>
</tr>
<tr>
<td>Coweeta, North Carolina</td>
<td>1:14,400</td>
<td>15.0</td>
<td>1,025</td>
<td>950</td>
<td>Oak Ridge National Laboratoryb</td>
</tr>
<tr>
<td>Salem, Oregon</td>
<td>1:250,000</td>
<td>92.9</td>
<td>1,031</td>
<td>1,186</td>
<td>USGSa</td>
</tr>
<tr>
<td>Nuremberg, Pennsylvania</td>
<td>1:24,000</td>
<td>30.0</td>
<td>371</td>
<td>405</td>
<td>USGSa</td>
</tr>
<tr>
<td>Walker Branch, Tennessee</td>
<td>1:1,200</td>
<td>3.0</td>
<td>320</td>
<td>97</td>
<td>Oak Ridge National Laboratoryb</td>
</tr>
<tr>
<td>Sleepers River, Vermont</td>
<td>1:24,000</td>
<td>30.0</td>
<td>441</td>
<td>621</td>
<td>USGSa</td>
</tr>
<tr>
<td>White Oak Run, Virginia</td>
<td>1:24,000</td>
<td>40.0</td>
<td>679</td>
<td>580</td>
<td>University of Virginiac</td>
</tr>
</tbody>
</table>

*b Oak Ridge National Laboratory, Oak Ridge, Tennessee.
*c University of Virginia, Charlottesville, Virginia.

during purposes only and does not constitute endorsement by the U.S. Geological Survey.

Computation of the In (a/tan β) Distribution

The DEMs were used to calculate a and tan β for each grid cell in the watershed. To do so required the calculation of the total area draining into each cell (A), as well as the contour length (C) and slope gradient (tan β) along which drainage from the cell occurs (a = A/C). Drainage from a cell was assumed to occur in only the steepest downslope direction for the sfd algorithm and was assumed to occur in all downslope directions for the mfd algorithm (Figure 1). Both the sfd and mfd algorithms required removal of local depressions in the elevation data. In this study, elevation data representing local depressions were filled using ARC/INFO GRID.

Single Flow Direction Algorithm

The single flow direction algorithm is based on the methods reported by Jenson and Domingue [1988]. These methods are incorporated into ARC/INFO GRID, which was used in this study to derive the ln (a/tan β) distribution. The steps to compute the ln (a/tan β) distribution using the sfd algorithm are as follows:

1. The elevation of each grid cell is compared to the elevations of its eight neighboring cells, and the steepest downslope direction is assigned to each cell in the grid. Directions also are assigned to grid cells in flat areas to provide a continuous path from all cells in the watershed to the outlet of the watershed. This is accomplished by "finding" the neighboring cell with the steepest downslope direction and assigning that direction to the flat area cell [Jenson and Domingue, 1988].

2. The number of grid cells upslope from each grid cell (number of upslope cells that drain into a given grid cell) is calculated and the total upslope area, A, is computed as:

   \[ A = (\text{number of upslope cells} + 1) \times (\text{grid cell area}) \]  

3. The magnitude of tan β in the steepest downslope direction is calculated as:

   \[ \tan \beta = (\text{change in elevation between neighboring grid cells}) + (\text{horizontal distance between centers of neighboring grid cells}) \]  

The change in elevation value is reset to 0.5 x (vertical resolution of elevation data) for flat areas where there is no downslope neighboring cell. The elevation data in many DEMs, for example, are given to the nearest whole meter; these DEMs therefore would have a vertical resolution of 1 m. Resetting the change in elevation value is required because it is assumed implicitly in TOPMODEL that all water in the watershed that does not evaporate eventually drains in some downslope direction.

4. The ln (a/tan β) values are calculated as:

   \[ \ln (a/tan \beta) = \ln \left( \frac{A}{C \tan \beta} \right) \]  

The contour length of the border between adjacent cells, C, is assumed to be equal to the grid cell length. Dividing the total upslope area, A, by C gives a, the area per unit contour length.

Multiple Flow Direction Algorithm

The multiple flow direction algorithm uses one set of computations for sloping areas in the watershed (grid cells with one or more downslope neighboring cells) and another set of computations for flat areas. The computations for sloping areas are

Figure 1. Plan views of an elevation grid showing differences in the contour length and downslope directions for the single flow direction (sfd) and multiple flow direction (mfd) algorithms.
similar to those reported by Wolock et al. [1989, 1990] and by Quinn et al. [1991]. The steps are as follows:

1. An initial value for \( A \) equal to the grid cell area is assigned to each grid cell. Also, flow directions are assigned to grid cells in flat areas using ARC/INFO GRID such that there is a continuous path from all cells in the watershed to the outlet of the watershed. This is accomplished by "finding" the neighboring cell with the steepest downslope direction and assigning that direction to the flat area cell [Jenson and Domingue, 1988].

2. Beginning with the grid cell having the highest elevation, the elevation of the cell is compared with that of its four diagonal and four cardinal neighboring cells. The contour length of the cell boundary with downslope neighboring cells, \( C \), is computed as

\[
C = \sum_{i=1}^{n} L_i, \tag{6}
\]

where \( L_i \) are the lengths of the cell boundaries between the cell of interest and its \( n \) downslope neighboring cells. The value of \( L_i \) is set to 0.6 \( \times \) (grid cell length) for boundaries with cardinal neighboring downslope cells and to 0.4 \( \times \) (grid cell length) for boundaries with diagonal neighboring downslope cells. These values of \( L_i \) were chosen so that the maximum value of \( C \) (the case of eight neighboring downslope cells) would equal the total boundary length between the cell of interest and all of its neighboring downslope cells. If the cell of interest is in a flat area and has no neighboring cells downslope, then \( C \) is assigned the value of the grid cell length.

3. The slope gradient in the downslope direction, \( \tan \beta \), is computed as

\[
\tan \beta = \left[ \sum_{i=1}^{n} \left( \tan \beta L_i \right) \right] \left( \sum_{i=1}^{n} L_i \right)^{-1}, \tag{7}
\]

where \( \tan \beta \) are the slope gradients between the cell of interest and its \( n \) downslope neighboring cells. If the cell of interest is in a flat area and has no downslope neighboring cells, then the slope gradient is computed as

\[
\tan \beta = \left( \frac{0.5 \times \text{vertical resolution of elevation data}}{ \text{(horizontal distance between centers of neighboring grid cells)}} \right). \tag{8}
\]

4. The \( \ln (a/\tan \beta) \) value then is computed as

\[
\ln (a/\tan \beta) = \ln \left[ A/(C \tan \beta) \right]. \tag{9}
\]

5. The total upslope area is distributed to the downslope neighboring cells as

\[
\Delta A_i = A (\tan \beta L_i) \left( \sum_{i=1}^{n} (\tan \beta L_i) \right)^{-1}. \tag{10}
\]

where \( \Delta A_i \) is the drainage area contributed from the cell of interest to the downslope neighboring cell \( i \). \( \Delta A_i \) is added to the previous value of \( A \) for that downslope neighboring cell. If the cell of interest is in a flat area and has no downslope neighboring cell, then the total upslope area is distributed to the single neighboring cell indicated by the direction determined in the first step.

6. The process continues by returning to step 2 and repeating steps 2–5 for the next lowest elevation grid cell. This step is continued until all grid cells have been processed.

Differences in the \( \ln (a/\tan \beta) \) Distributions Computed With the Single and Multiple Flow Direction Algorithms

Figure 2 shows gray-shaded maps of \( \ln (a) \), \( \ln (1/\tan \beta) \), and \( \ln (a/\tan \beta) \) spatial distributions computed using the sfd and mfd algorithms for a 70-column by 100-row section of the Salem, Oregon, DEM. Values of \( \ln (a) \) measure the amount of upslope drainage area and local flow convergence or divergence; values of \( \ln (1/\tan \beta) \) measure the local gravitational gradient. Values of \( \ln (a/\tan \beta) \) are highest (darkest gray shades) where \( \ln (a) \) is highest (large upslope areas and flow convergence) and \( \ln (1/\tan \beta) \) is highest (gentle slopes).

The spatial distribution of \( \ln (1/\tan \beta) \) values generated by the sfd algorithm looks identical to the distribution produced by the mfd algorithm (Figure 2b). The spatial distributions of \( \ln (a) \) and \( \ln (a/\tan \beta) \) values, however, look somewhat different (Figures 2a and 2c). The sfd algorithm produces a spatial distribution with a more discrete, rougher pattern, and the mfd algorithm produces a distribution with a smoother pattern. These results agree with those of Quinn et al. [1991], who showed that the spatial distribution patterns of upslope drainage area and of \( \ln (a/\tan \beta) \) values were smoother for their mfd algorithm than for their sfd algorithm. Also, Moore [1995] showed that the spatial distribution pattern of upslope area per unit contour length was more linear and less smooth for the sfd algorithm than the pattern derived from the mfd algorithm. The smoothness in the pattern of the spatial distribution computed using the mfd algorithm occurs because the upslope area is partitioned to multiple downslope neighboring cells (equation (10)).

Recently, Costa-Cabral and Burges [1994] compared the spatial distribution patterns of upslope drainage area computed with sfd and mfd algorithms for idealized geometric surfaces. They showed that the mfd algorithm produced more accurate patterns of upslope drainage area compared to the sfd algorithm. They also demonstrated that even the mfd algorithm was prone to error under some conditions, and they described an alternative method to compute upslope drainage area based on aspect angles.

The spatial distributions of \( \ln (a) \), \( \ln (1/\tan \beta) \), and \( \ln (a/\tan \beta) \) values for the Salem, Oregon, DEM can be described by their mean, variance, and skew values (Table 2). Compared to the sfd algorithm, using the mfd algorithm results in an \( \ln (a/\tan \beta) \) distribution with a higher mean value and lower variance and skew values. The higher mean \( \ln (a/\tan \beta) \) value is associated with higher mean values of the \( \ln (a) \) and \( \ln (1/\tan \beta) \) distributions for the mfd algorithm compared to the sfd algorithm. Similarly, the lower variance and skew values of the \( \ln (a/\tan \beta) \) distribution are associated with lower variance and skew values of the \( \ln (a) \) and \( \ln (1/\tan \beta) \) distributions for the mfd algorithm compared to the sfd algorithm.

The lower mean \( \ln (1/\tan \beta) \) value for the sfd algorithm occurs because the sfd algorithm uses the steepest downslope direction to compute \( \tan \beta \) (equation (4)), whereas the mfd algorithm uses a weighted average of all downslope directions (equation (7)). The lower variance and skew \( \ln (1/\tan \beta) \) values for the mfd algorithm occur because using weighted-average values reduces cell-to-cell differences.

The higher mean \( \ln (a) \) value for the mfd algorithm may
Figure 2. (a) Values of $\ln(a)$, (b) values of $\ln(1/\tan\beta)$, and (c) values of $\ln(a/\tan\beta)$ computed using single flow direction (sfd, left panels) and multiple flow direction (mfd, right panels) algorithms for a 70-column by 100-row section of the Salem, Oregon, DEM. The location of the lower left corner of the DEM is 44°30'0"N latitude and 122°30'0"W longitude, and the grid cell size of the DEM is 93 by 93 m.

Figure 2 seem to be a counterintuitive result for the following reasons. The mfd algorithm allows for flow convergence (several cells draining into one downslope neighboring cell) and flow divergence (one cell draining into multiple downslope neighboring cells). The sfd algorithm, in contrast, allows only flow convergence. When flow convergence occurs, the upslope area ($A$) is concentrated in a downslope neighboring cell, thereby increasing the value of upslope area per unit contour length, $a = \frac{a}{\tan\beta}$.
Table 2. Statistics of \( \ln (a/\tan(\beta)) \), \( \ln (a) \), and \( \ln (1/\tan(\beta)) \) Distributions Computed for the Salem, Oregon, DEM Using the Single Flow Direction (sfd) and Multiple Flow Direction (mfd) Algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Variable</th>
<th>Mean</th>
<th>Variance</th>
<th>Skew</th>
</tr>
</thead>
<tbody>
<tr>
<td>sfd</td>
<td>( \ln (a/\tan(\beta)) )</td>
<td>7.18</td>
<td>6.87</td>
<td>32.70</td>
</tr>
<tr>
<td></td>
<td>( \ln (a) )</td>
<td>5.66</td>
<td>3.41</td>
<td>9.40</td>
</tr>
<tr>
<td>mfd</td>
<td>( \ln (a/\tan(\beta)) )</td>
<td>7.81</td>
<td>6.45</td>
<td>30.31</td>
</tr>
<tr>
<td></td>
<td>( \ln (a) )</td>
<td>5.87</td>
<td>3.18</td>
<td>8.97</td>
</tr>
<tr>
<td></td>
<td>( \ln (1/\tan(\beta)) )</td>
<td>1.94</td>
<td>1.16</td>
<td>2.22</td>
</tr>
</tbody>
</table>

Comparison of TOPMODEL Simulations Using Single and Multiple Flow Direction Algorithms

TOPMODEL was applied to Sleepers River, Vermont, W-3 watershed to determine the effects on hydrologic predictions of differences in the \( \ln (a/\tan(\beta)) \) distribution statistics computed using the sfd and mfd algorithms. The 30-m-resolution DEM was used to compute the \( \ln (a/\tan(\beta)) \) distribution using the sfd and mfd algorithms (Table 4). TOPMODEL was run using these two sets of \( \ln (a/\tan(\beta)) \) statistics and a 2-year time series of daily precipitation and temperature data.

In the first simulation the \( \ln (a/\tan(\beta)) \) statistics computed from the mfd algorithm were used, and the subsurface hydraulic parameters, \( f \) and \( K_o \) in (1) and (2), were adjusted to optimize the fit between the predicted streamflow and the observed streamflow during the second year of the simulation. (The first year of the simulation was used as a “warm-up” period to minimize the effects of arbitrary initial conditions on model predictions.) The Rosenbrock optimization algorithm [Hornberger et al., 1985] was used to automatically adjust the subsurface hydraulic parameters to maximize the model efficiency, which would assume a value of 100% for a perfect fit. The percentage of simulated overland flow in total flow was recorded for the second year of the simulation as an indicator of the dominant streamflow generation mechanism simulated by the model.

Agreement between the predicted and observed streamflow was good when the \( \ln (a/\tan(\beta)) \) statistics for the mfd algorithm
Figure 3. (a) Mean, (b) variance, and (c) skew of In (a/tan β) distributions computed using the single flow direction (sfd) and multiple flow direction (mfd) algorithms for watersheds delineated from DEMs in Arizona, Colorado, Louisiana, Nebraska, Oregon, Pennsylvania, and Vermont.

Figure 4. Relation between DEM resolution and the (a) mean, (b) variance, and (c) skew of the In (a/tan β) distribution computed using single flow direction (sfd) and multiple flow direction (mfd) algorithms for DEMs in Coweeta, North Carolina, Walker Branch, Tennessee, and Sleepers River, Vermont.
were used and the subsurface hydraulic parameters were calibrated (Table 5). The model efficiency was 89%, and the percentage of overland flow in total flow was 26%. This percentage of overland flow in total flow indicates that during the second year of the simulation, 26% of the total streamflow volume originated as overland flow and 74% originated as subsurface flow.

In the second simulation the In \((a/tan/3)\) statistics computed using the sfd algorithm were substituted, and the model was run without recalibration; the optimal subsurface hydraulic parameters from the calibrated mfd simulation were used without readjustment. The model efficiency for this simulation during the same 2-year period was 84%, a slightly poorer fit compared to the calibrated mfd simulation (Table 5).

The percentage of overland flow in total flow was 20% for the second simulation (Table 5). This result shows that the simulated streamflow generation mechanism was affected by the In \((a/tan/3)\) distribution; the only difference between the first and second simulations was the In \((a/tan/3)\) statistics. The mean In \((a/tan/3)\) value was lower, and the variance and skew values were higher, for the sfd algorithm compared to the mfd algorithm (Table 4). Previous work [Wolock et al., 1990] has shown that an increase in the mean In \((a/tan/3)\) can or variance value causes an increase in the percentage of overland flow in total flow. An increase in the mean In \((a/tan/3)\) value causes a decrease in the subsurface flow rate (see (2)). This decrease in the subsurface flow rate decreases the watershed average depth to the water table, which increases the likelihood that saturated land surfaces areas will develop and produce overland flow (see (1)). Considered alone, an increase in the In \((a/tan/3)\) variance also tends to increase the extent of saturated areas predicted in (1). The results shown in Table 5 indicate that when the In \((a/tan/3)\) distribution from the sfd algorithm with lower mean and higher variance is used, the net effect is a small decrease in the percentage of overland flow in total flow.

In the third simulation the In \((a/tan/3)\) values from the sfd algorithm were used, and the subsurface hydraulic parameters were adjusted to optimize the model efficiency, which was 88% after calibration (Table 5). Calibrating TOPMODEL increased the percentage of overland flow in total flow from 20% for the uncalibrated sfd simulation to 25% for the calibrated sfd simulation. During the automatic calibration, \(K_0\) decreased from 167 to 82 \(\text{mm/d}\), and \(f\) increased from 3.3 to 3.5 \(\text{m}^{-1}\). The main effect of these changes in the parameter values was to represent the soil as less transmissive, which increased the amount of overland flow simulated by the model. After calibration, results from the sfd and mfd algorithms were nearly identical; they had equally good fits to the observed streamflow data (model efficiencies of 88% and 89%, respectively) and almost the same simulated flow paths (percentages of overland flow in total flow of 25% and 26%, respectively). These results show therefore that if TOPMODEL is calibrated, the sfd and mfd algorithms produce virtually identical results in terms of model fit to observed streamflow data and streamflow generation mechanisms. Using the different In \((a/tan/3)\) computation algorithms, however, causes the optimization routine to “find” very different subsurface hydraulic parameters that maximize the agreement between the observed and simulated streamflow. Thus TOPMODEL simulates the same streamflow generation mechanisms but compensates for different topographic parameters by changing subsurface hydraulic parameters.

It may be argued that the mfd and sfd algorithms can be evaluated according to the “reasonableness” of the optimal subsurface hydraulic parameters; the In \((a/tan/3)\) computation algorithm that is associated with the most realistic optimal subsurface hydraulic parameters would be the superior algorithm. Unfortunately, it is difficult to know what are reasonable or realistic values for \(K_0\) and \(f\). These parameters, like many subsurface hydraulic parameters, are extremely variable over space and difficult to meaningfully quantify, particularly at the watershed scale [Beven, 1989].

TOPMODEL also was run using the In \((a/tan/3)\) distributions computed with the sfd and mfd algorithms using the Sleepers River, Vermont, DEM resampled at 60- and 90-m grid cell sizes to determine if the results were affected by DEM resolution. The subsurface hydraulic parameters \(K_0\) and \(f\) were adjusted to optimize the agreement between the simulated and observed streamflow. Although the In \((a/tan/3)\) statistics were different because of the different resolution data and the different computational algorithms (Table 6), after calibration the model efficiencies and simulated flow paths were almost equal.

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### Table 4. Statistics of the In \((a/tan/3)\) Distribution for Sleepers River, Vermont, Watershed W-3 Computed Using Single Flow Direction (sfd) and Multiple Flow Direction (mfd) Algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Mean</th>
<th>Variance</th>
<th>Skew</th>
</tr>
</thead>
<tbody>
<tr>
<td>sfd</td>
<td>6.56</td>
<td>6.12</td>
<td>23.59</td>
</tr>
<tr>
<td>mfd</td>
<td>7.30</td>
<td>5.67</td>
<td>19.23</td>
</tr>
</tbody>
</table>

### Table 5. TOPMODEL Simulation Results for Sleepers River, Vermont, Watershed W-3 Using In \((a/tan/3)\) Distributions Computed Using Single Flow Direction (sfd) and Multiple Flow Direction (mfd) Algorithms

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Model Efficiency, %</th>
<th>(K_0), mm/d</th>
<th>(f), 1/m</th>
<th>Percentage of Overland Flow in Total Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated mfd</td>
<td>89</td>
<td>167</td>
<td>3.3</td>
<td>26</td>
</tr>
<tr>
<td>Uncalibrated sfd</td>
<td>84</td>
<td>167</td>
<td>3.3</td>
<td>20</td>
</tr>
<tr>
<td>Calibrated sfd</td>
<td>88</td>
<td>82</td>
<td>3.5</td>
<td>25</td>
</tr>
</tbody>
</table>

**TOPMODEL subsurface hydraulic parameters, \(f\) (the parameter related to the rate of change of saturated hydraulic conductivity with depth) and \(K_0\) (the maximum saturated hydraulic conductivity), were optimized for the calibrated mfd simulation. The same subsurface hydraulic parameter values were used for the uncalibrated sfd simulation and then reoptimized for the calibrated sfd simulation. The model efficiency is the percentage of the variance in the measured streamflow data explained by the model.**

### Table 6. Statistics of the In \((a/tan/3)\) Distribution for Sleepers River, Vermont, Watershed W-3 Computed Using the Single Flow Direction (sfd) and Multiple Flow Direction (mfd) Algorithms at 30-, 60-, and 90-m DEM Resolution

<table>
<thead>
<tr>
<th>DEM Resolution, m</th>
<th>Algorithm</th>
<th>Mean</th>
<th>Variance</th>
<th>Skew</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>sfd</td>
<td>6.56</td>
<td>6.12</td>
<td>23.59</td>
</tr>
<tr>
<td></td>
<td>mfd</td>
<td>7.30</td>
<td>5.67</td>
<td>19.23</td>
</tr>
<tr>
<td>60</td>
<td>sfd</td>
<td>7.31</td>
<td>6.34</td>
<td>25.11</td>
</tr>
<tr>
<td></td>
<td>mfd</td>
<td>8.02</td>
<td>5.94</td>
<td>21.01</td>
</tr>
<tr>
<td>90</td>
<td>sfd</td>
<td>7.73</td>
<td>6.26</td>
<td>25.23</td>
</tr>
<tr>
<td></td>
<td>mfd</td>
<td>8.41</td>
<td>6.00</td>
<td>21.69</td>
</tr>
</tbody>
</table>
identical (Table 7). Adjusting the subsurface hydraulic parameters compensated for the change in the In \((a/\tan \beta)\) distribution caused by DEM resolution and the In \((a/\tan \beta)\) computation algorithm.

Previous studies have shown that DEM resolution affects computation of the In \((a/\tan \beta)\) distribution and subsequent TOPMODEL predictions [Wolock and Price, 1994; Zhang and Montgomery, 1994]. In both of these previous studies the subsurface hydraulic parameters were not recalibrated after using different In \((a/\tan \beta)\) distributions from various DEM resolutions. The results reported here (Table 7), however, show that adjusting the subsurface hydraulic parameters through model calibration compensates for the effects of changing DEM resolution on the In \((a/\tan \beta)\) distribution.

TOPMODEL also was applied to the White Oak Run, Virginia, watershed to determine if the results obtained for Sleepers River, Vermont, were similar to those obtained for Sleepers River, Vermont. As before, TOPMODEL was run using these two sets of topographic parameters and a 2-year time series of daily precipitation and temperature data.

Results for the three simulations (Table 9) for White Oak Run, Virginia, were similar to those obtained for Sleepers River, Vermont. The results for the two watersheds both indicate, therefore, that when TOPMODEL is used without calibration, using the sdf and mfd algorithms to derive the In \((a/\tan \beta)\) distribution causes the model to simulate slightly different flow paths; a higher percentage of overland flow in total flow is simulated when the mfd algorithm is used compared to when the sdf algorithm is used. If, however, TOPMODEL is calibrated by adjusting subsurface hydraulic parameters, then the sdf and mfd algorithms give nearly identical results in terms of model fit to observed data and simulated flow paths.

Conclusions

Use of sdf and mfd algorithms caused differences in the spatial and statistical distributions of In \((a/\tan \beta)\) values computed from DEMs. The In \((a/\tan \beta)\) distribution computed with the sdf algorithm had a lower mean and higher variance and skew. The differences were consistent for DEMs of various grid cell resolution from diverse topographic settings and for watersheds of different sizes. TOPMODEL model efficiency and simulated flow paths were affected only slightly when the In \((a/\tan \beta)\) distribution is computed with the sdf algorithm instead of the mfd algorithm. Any difference in the model efficiency and simulated flow paths between the sdf and mfd algorithms essentially disappeared, however, when the model was calibrated by adjusting subsurface hydraulic parameters.

Recently, Moore [1995] concluded that the sdf algorithm was inferior to the mfd algorithm for deriving hydrologically meaningful topographic indices. Research described herein suggests that although the spatial and statistical distributions of In \((a/\tan \beta)\) values are affected by the computational algorithm, the aggregated effects on TOPMODEL model efficiency and simulated flow paths are minimal. The choice of algorithm, however, will affect the simulated spatial distribution of hydrologic characteristics such as soil moisture content. Therefore, if TOPMODEL is being used to simulate spatial patterns of

### Table 7. TOPMODEL Simulation Results for Sleepers River, Vermont, Watershed W-3 Using In \((a/\tan \beta)\) Distributions Computed With Single Flow Direction (sfd) and Multiple Flow Direction (mfd) Algorithms at 30-, 60-, and 90-m DEM Resolutions

<table>
<thead>
<tr>
<th>DEM Resolution, m</th>
<th>Simulation</th>
<th>Model Efficiency, (%)</th>
<th>(f), 1/m</th>
<th>(K_o), mm/d</th>
<th>Percentage of Overland Flow in Total Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>calibrated mfd</td>
<td>89</td>
<td>3.3</td>
<td>167</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>calibrated sdf</td>
<td>88</td>
<td>3.5</td>
<td>82</td>
<td>25</td>
</tr>
<tr>
<td>60</td>
<td>calibrated mfd</td>
<td>88</td>
<td>3.5</td>
<td>319</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>calibrated sdf</td>
<td>89</td>
<td>3.3</td>
<td>177</td>
<td>25</td>
</tr>
<tr>
<td>90</td>
<td>calibrated mfd</td>
<td>88</td>
<td>3.6</td>
<td>441</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>calibrated sdf</td>
<td>89</td>
<td>3.3</td>
<td>271</td>
<td>25</td>
</tr>
</tbody>
</table>

TOPMODEL subsurface hydraulic parameters, \(f\) (the parameter related to the rate of change of saturated hydraulic conductivity with depth) and \(K_o\) (the maximum saturated hydraulic conductivity), were optimized for both the calibrated mfd and sdf simulations. The model efficiency is the percentage of the variance in the measured streamflow data explained by the model.

### Table 8. Statistics of the In \((a/\tan \beta)\) Distribution for White Oak Run, Virginia, Watershed Computed Using Single Flow Direction (sfd) and Multiple Flow Direction (mfd) Algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Mean</th>
<th>Variance</th>
<th>Skew</th>
</tr>
</thead>
<tbody>
<tr>
<td>sdf</td>
<td>6.08</td>
<td>5.63</td>
<td>23.33</td>
</tr>
<tr>
<td>mfd</td>
<td>6.64</td>
<td>5.63</td>
<td>22.61</td>
</tr>
</tbody>
</table>

Conclusions

Use of sdf and mfd algorithms caused differences in the spatial and statistical distributions of In \((a/\tan \beta)\) values computed from DEMs. The In \((a/\tan \beta)\) distribution computed with the sdf algorithm had a lower mean and higher variance and skew. The differences were consistent for DEMs of various grid cell resolution from diverse topographic settings and for watersheds of different sizes. TOPMODEL model efficiency and simulated flow paths were affected only slightly when the In \((a/\tan \beta)\) distribution is computed with the sdf algorithm instead of the mfd algorithm. Any difference in the model efficiency and simulated flow paths between the sdf and mfd algorithms essentially disappeared, however, when the model was calibrated by adjusting subsurface hydraulic parameters.

Recently, Moore [1995] concluded that the sdf algorithm was inferior to the mfd algorithm for deriving hydrologically meaningful topographic indices. Research described herein suggests that although the spatial and statistical distributions of In \((a/\tan \beta)\) values are affected by the computational algorithm, the aggregated effects on TOPMODEL model efficiency and simulated flow paths are minimal. The choice of algorithm, however, will affect the simulated spatial distribution of hydrologic characteristics such as soil moisture content. Therefore, if TOPMODEL is being used to simulate spatial patterns of
hydrologic characteristics, then the mfd algorithm may be preferred to the sfd algorithm; determination of the superior algorithm will require comparison of simulated and measured spatial patterns of hydrologic characteristics. If the model is being used to simulate streamflow or to partition streamflow into its flow path components, however, the choice of In (a/tan β) computational algorithm will not make a difference.

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